Identification of planetary signals with posterior samplings

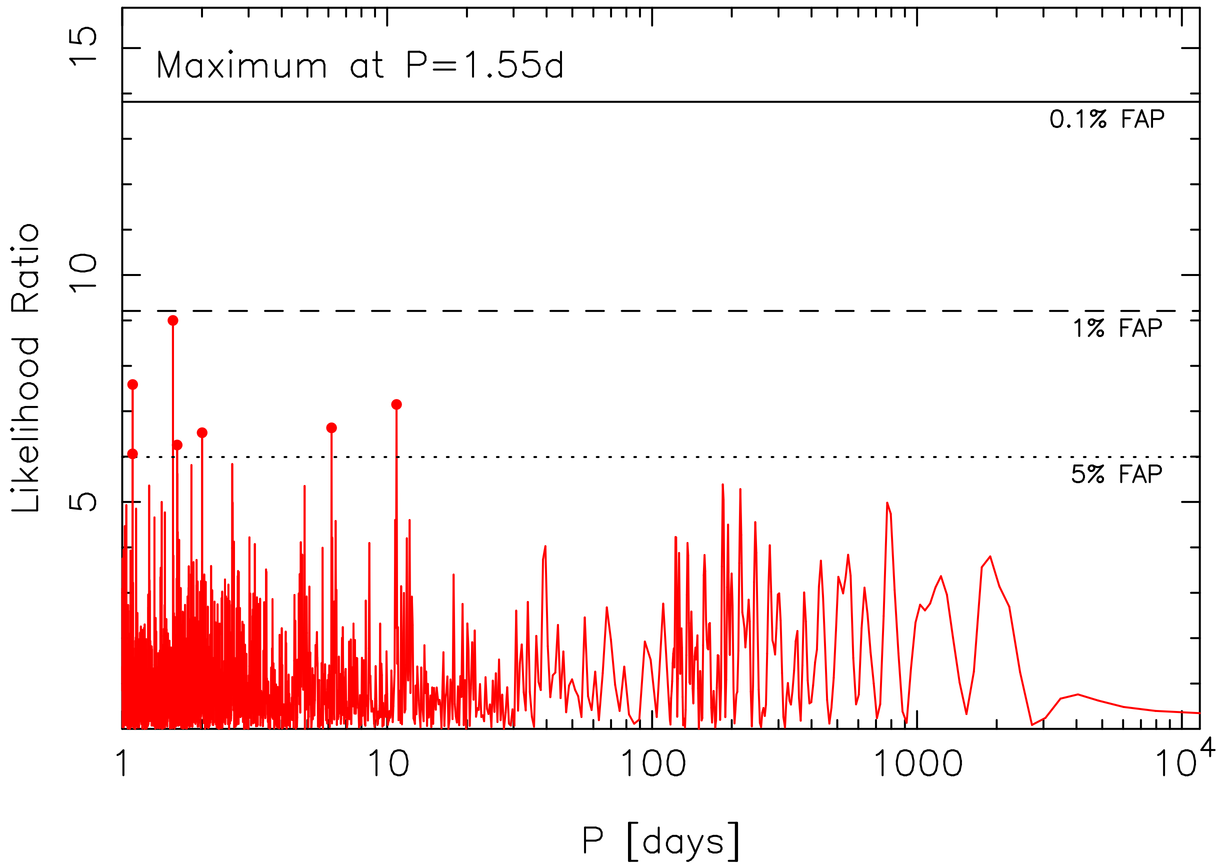
Mikko Tuomi et al.

University of Hertfordshire, Centre for Astrophysics Research

Email: m.tuomi@herts.ac.uk

Presentation, 2.2.2017

Why have another technique when periodograms already exist? Right?



Do periodograms work? Forget about residual analyses.

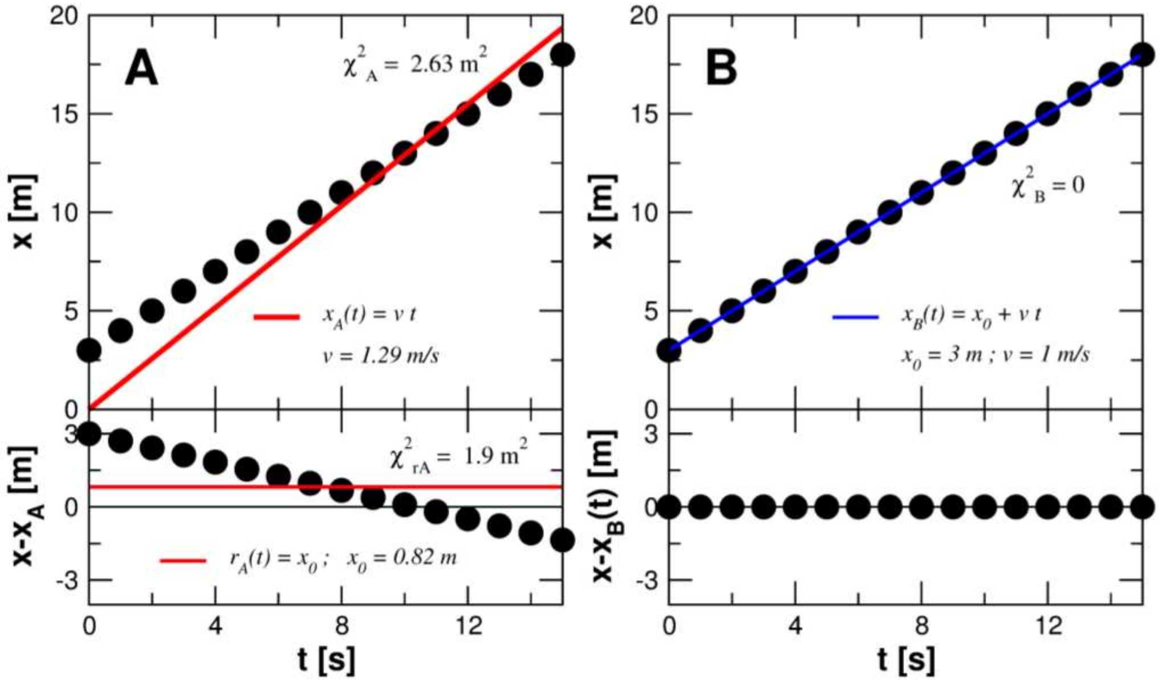


Fig. 1: This example illustrates why residual statistics must not be used to assess significances in multi-parametric fits to data. We want to know whether a constant x_0 is needed to model the position x of a body

Searching for one signal is possible.

1. Several data sets that need to be modelled with non-linear models.
2. Searches for several signals.
3. The problem becomes highly multimodal and high-dimensional.

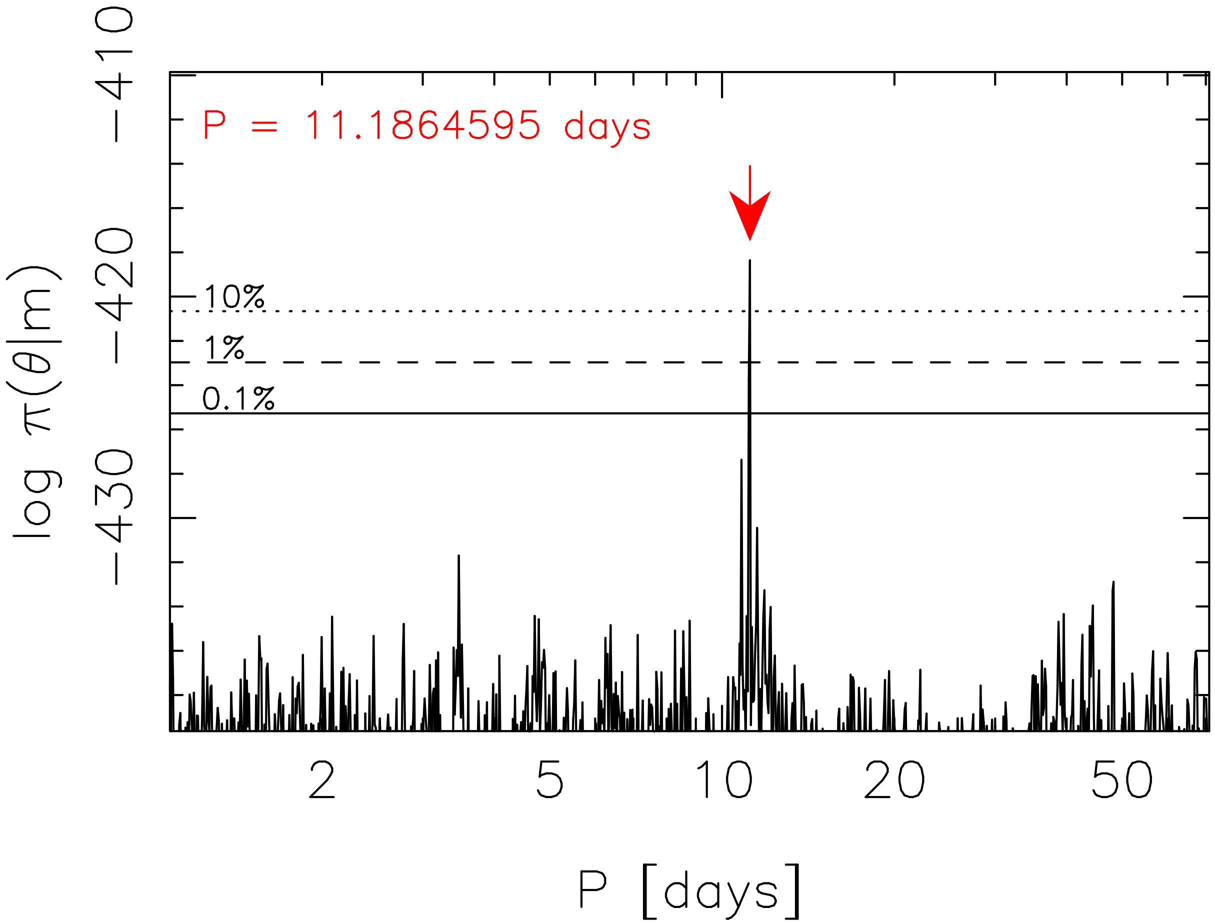
A typical statistical model for RV time-series:

$$m_{i,l} = \gamma_l + \dot{\gamma} t_i + f_k(t_i) + \epsilon_{i,l} + \sum_{j=1}^{q} c_{j,l} \xi_{j,i,l} + \sum_{j=1}^{p} \phi_{j,l} \exp \left\{ \frac{t_{i-j} - t_i}{\tau_l} \right\} r_{i-j,l}, \tag{1}$$

where

$$f_k(t_i) = \sum_{j=1}^{k} K_j \Big[\cos(\omega_j + \nu_j(t_i, P_j, M_{0,j})) + e_j \cos(\omega_j) \Big] \tag{2}$$

MCMC samplings can do the trick.



MCMC samplings can do the trick.

1. Bonfils et al. (2013) reported three planet candidates orbiting GJ 163. This was based on 153 HARPS RVs.

MCMC samplings can do the trick.

1. Bonfils et al. (2013) reported three planet candidates orbiting GJ 163. This was based on 153 HARPS RVs.

2. In the same issue of A&A, Tuomi & Anglada-Escudé independently reported the same planet candidates with only 55 of the HARPS RVs available.

MCMC samplings can do the trick.

1. Bonfils et al. (2013) reported three planet candidates orbiting GJ 163. This was based on 153 HARPS RVs.

2. In the same issue of A&A, Tuomi & Anglada-Escudé independently reported the same planet candidates with only 55 of the HARPS RVs available.

3. Thanks to fantastic ESO archives the data was available!

MCMC samplings can do the trick.

1. Bonfils et al. (2013) reported three planet candidates orbiting GJ 163. This was based on 153 HARPS RVs.

2. In the same issue of A&A, Tuomi & Anglada-Escudé independently reported the same planet candidates with only 55 of the HARPS RVs available.

3. Thanks to fantastic ESO archives the data was available!

4. Same result based on only $\sim 35\%$ of the data – what made the difference?

MCMC samplings can do the trick.

1. Bonfils et al. (2013) reported three planet candidates orbiting GJ 163. This was based on 153 HARPS RVs.

2. In the same issue of A&A, Tuomi & Anglada-Escudé independently reported the same planet candidates with only 55 of the HARPS RVs available.

3. Thanks to fantastic ESO archives the data was available!

4. Same result based on only $\sim$ 35% of the data – what made the difference?

5. Contribution from TERRA algorithm (Anglada-Escudé & Butler, 2012) but mostly due to better sensitivity due to Bayesian approach, and MCMC samplings.

MCMC samplings can do the trick.

1. Bonfils et al. (2013) reported three planet candidates orbiting GJ 163. This was based on 153 HARPS RVs.

2. In the same issue of A&A, Tuomi & Anglada-Escudé independently reported the same planet candidates with only 55 of the HARPS RVs available.

3. Thanks to fantastic ESO archives the data was available!

4. Same result based on only $\sim 35\%$ of the data – what made the difference?

5. Contribution from TERRA algorithm (Anglada-Escudé & Butler, 2012) but mostly due to better sensitivity due to Bayesian approach, and MCMC samplings.

6. Most importantly: Tuomi & Anglada-Escudé were not limited by residual analyses.

Simple MCMC algorithm

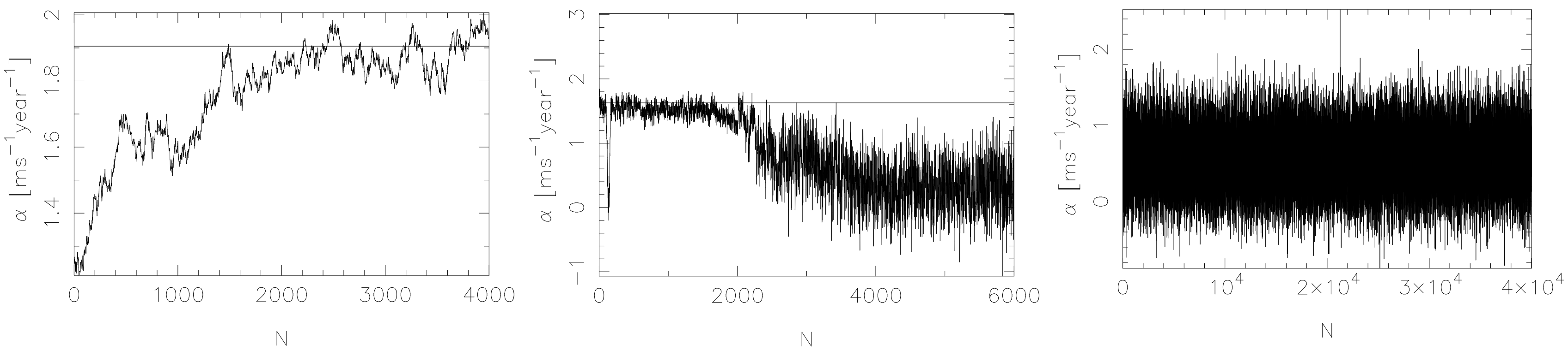
1. Choose an initial state in the parameter space $\theta \in \Omega$.
2. Choose proposal density.
3. Draw a vector θ' from the proposal density.
4. Test whether θ' is accepted.
5. If θ' is accepted, set $\theta_i = \theta'$ – otherwise set $\theta_i = \theta_{i-1}$.
6. Go to 3.

Adaptive Metropolis algorithm.

Adapting proposal density makes it possible to use the information already gathered about the posterior to enable faster “convergence“.

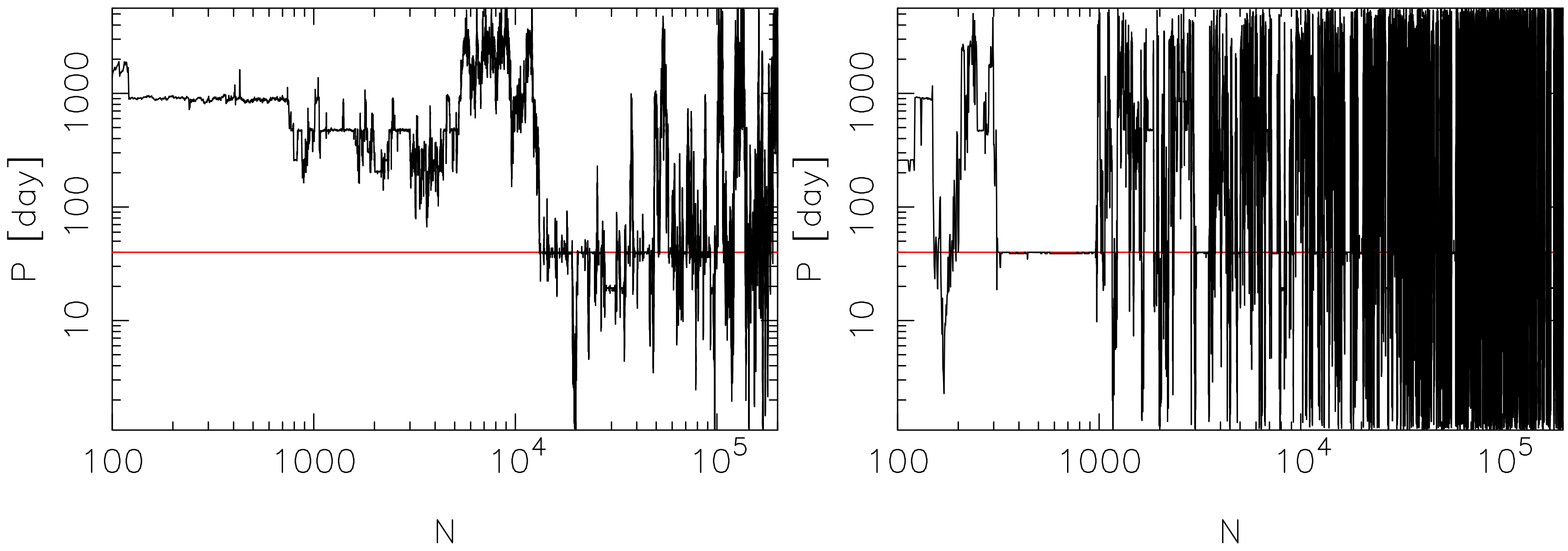
Assuming Gaussian proposal density, the covariance is updated according to

$$C_{n+1} = \frac{n+1}{n} C_n + \frac{s}{n} \Big[n\bar{\theta}_{n-1}\bar{\theta}_{n-1}^T - (n+1)\bar{\theta}_n\bar{\theta}_n^T + \theta_n\theta_n^T + \epsilon I \Big], \quad (3)$$



Delayed rejection AM (DRAM) algorithm.

Basic idea: if a proposed vector is rejected, do not give up but propose another one.



Improving DRAM samplings.

Assume there is a high maximum such that the chain quickly identifies it and gets stuck in it.

Solution: sample $\pi(\theta)^\beta$ rather than $\pi(\theta)$, where $\beta \in (1, 0)$.

This is suitable for signal searches because the transformation retains the positions of the maxima.

Parameter estimation must be carried out by setting $\beta = 1$.

Improving DRAM samplings.

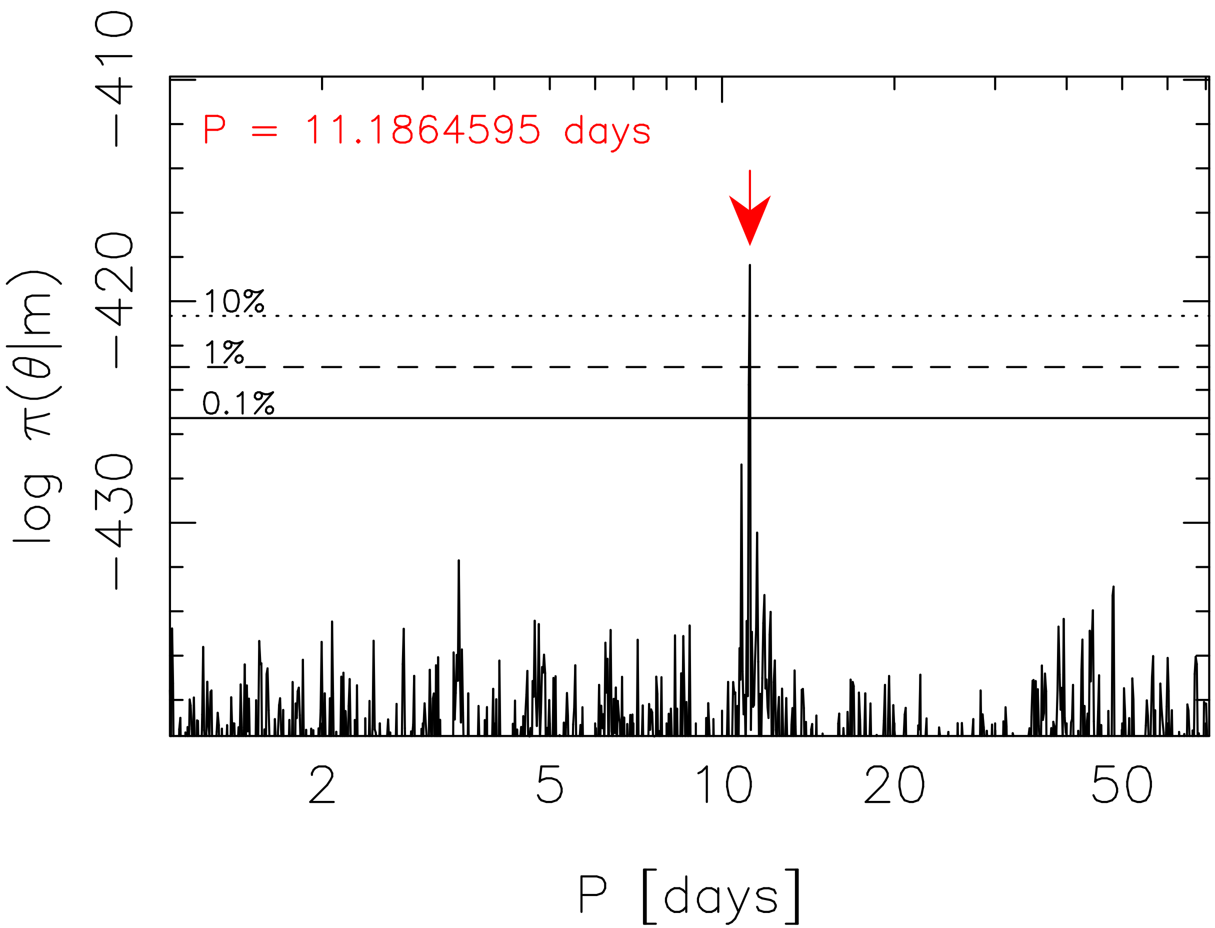
Reversible jump modification (Green 1995).

Local optimization.

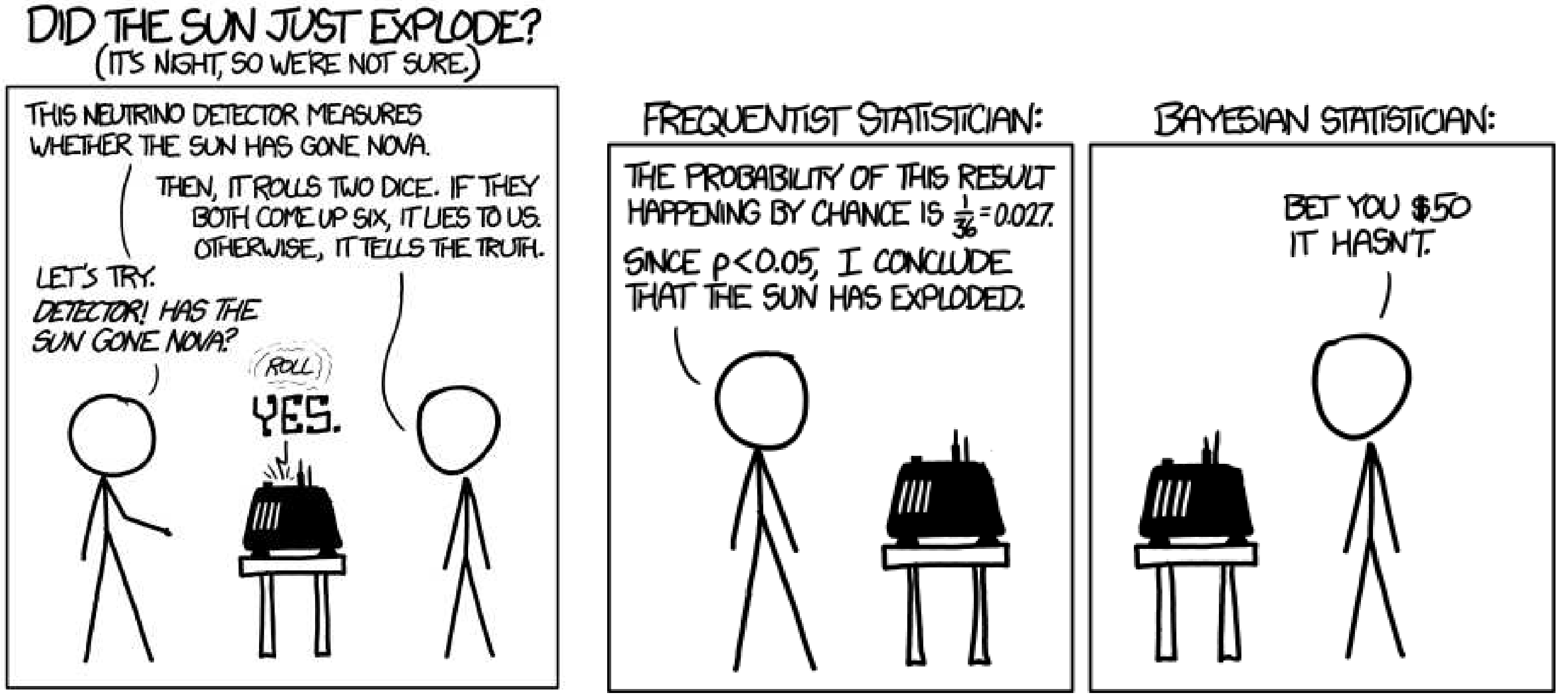
Parallelization with multiple chains.

Dozens of algorithms exist, each fine-tuned to solve a specific statistical problem.

MCMC samplings can do the trick.



Philosophy


DID THE SUN JUST EXPLODE?
(IT'S NIGHT, SO WE'RE NOT SURE.)
THIS NEUTRINO DETECTOR MEASURES WHETHER THE SUN HAS GONE NOVA.
THEN, IT ROLLS TWO DICE. IF THEY BOTH COME UP SIX, IT LIES TO US. OTHERWISE, IT TELLS THE TRUTH.
LET'S TRY.
DETECTOR! HAS THE SUN GONE NOVA?
ROLL
YES.
FREQUENTIST STATISTICIAN:
THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS 1/36 = 0.027.
SINCE p<0.05, I CONCLUDE THAT THE SUN HAS EXPLODED.
BAYESIAN STATISTICIAN:
BET YOU $50 IT HASN'T.

Priors make a difference

All parameter values cannot be considered equally probable *a priori*.